

# Properties of $^8\text{Be}$ and $^{12}\text{C}$ deduced from the folding-potential model

P. Mohr, H. Abele, V. Kölle, G. Staudt

*Physikalisches Institut, Univ. Tübingen, D-72076 Tübingen, Germany*

H. Oberhummer and H. Krauss

*Institut für Kernphysik, Technische Universität Wien, A-1040 Vienna, Austria*

**Abstract:** The  $\alpha$ - $\alpha$  differential cross sections are analyzed in the optical model using a double-folded potential. With the knowledge of this potential bound and resonance-state properties of  $\alpha$ -cluster states in  $^8\text{Be}$  and  $^{12}\text{C}$  as well as astrophysical S-factors of  $^4\text{He}(\alpha, \gamma)^8\text{Be}$  and  $^8\text{Be}(\alpha, \gamma)^{12}\text{C}$  are calculated.  $\Gamma_\gamma$ -widths and B(E2)-values are deduced.

**PACS:** 25.55.Ci, 25.70.Ef, 24.10.Ht

In order to determine reaction rates for the triple-alpha process [1], we have calculated in the potential model the  $\alpha$ - $\alpha$  differential cross section as well as the bound and resonance-state properties of  $\alpha$ -cluster states in  $^8\text{Be}$  and  $^{12}\text{C}$ . The potentials are calculated using the folding procedure [2], [3].

In this approach the nuclear densities are derived from nuclear charge distributions [4] and folded with an energy and density dependent NN-interaction  $v_{\text{eff}}$  [2]. By means of a normalization factor  $\lambda$  the depth of the potential can be adjusted to elastic scattering data and to bound and resonant state energies of nuclear cluster states. In this work the folding potentials were determined using the computer code DFOLD [5]. The imaginary part of the optical potential can be neglected at the low energies considered.

Using the folding procedure for the  $\alpha$ - $\alpha$  scattering potential we fitted the experimental differential cross section for all projectile energies below the break-up threshold ( $E_{\text{lab}} \approx 7\text{--}35\text{ MeV}$ ) with a renormalization factor  $\lambda = 1.656$ . An excellent agreement between the calculated and experimental cross section has been found [6]. With the same potential we reproduce the resonance energies and widths of the  $^8\text{Be}$  ground-state:  $E_{\text{R}}^{\text{th}} = 92.1\text{ keV}$  and of the  $2^+$ - and  $4^+$ -state in  $^8\text{Be}$  (see Table 1). Especially for the ground state the calculated values agree excellently with the experimental data. This result confirms that this nucleus can be well described in a two-cluster approach [7]. This marked  $\alpha$ -cluster structure is also reflected by the values of the  $\alpha$ -particle spectroscopic-amplitudes for these  $^8\text{Be}$ -states [8]:  $S(\text{g.s.})=1.48$ ,  $S(2^+)=1.46$ , and  $S(4^+)=1.39$ .

The s-wave function for the relative motion of the two  $\alpha$ -clusters in the  $^8\text{Be}$  ground state can be folded with the nucleon densities of both  $\alpha$ -particles to calculate the  $^8\text{Be}$  density [6]. In our calculation we used an upper cut-off radius of 20 fm. Averaging over  $4\pi$  we obtain the nucleon density distribution. The result is shown in Fig. 1. With the knowledge of the  $^8\text{Be}$  density distribution we are able

**Table 1.** Relevant data of resonant states in  $^8\text{Be}$ 

$J^\Pi$	$E^{\text{calc}}$ [MeV]	$E^{\text{exp}}$ [MeV] [9]	$\Gamma_\alpha^{\text{calc}}$ [eV]	$\Gamma_\alpha^{\text{exp}}$ [eV] [9]
$0^+$	0.0921	0.09212(5)	6.86	$6.8 \pm 1.7$
$2^+$	2.78	3.13(3)	$1.3 \cdot 10^6$	$1.5(2) \cdot 10^6$
$4^+$	11.02	11.5(3)	$\gtrsim 3 \cdot 10^6$	$\approx 3.5 \cdot 10^6$

**Table 2.** Relevant data of bound and resonant states in  $^{12}\text{C}$ 

$J^\Pi$	$E$ [MeV]	$\lambda$	$\Gamma_\alpha^{\text{calc}}$ [eV]	$\Gamma_\alpha^{\text{exp}}$ [eV] [10]
$0_1^+$	0.0	1.144	—	—
$2_1^+$	4.437	1.010	—	—
$0_2^+$	7.654	1.286	7.5	$8.3 \pm 1.0$
$2_2^+$	10.3	1.152	$\approx 2.0 \cdot 10^6$	$(3.0 \pm 0.7) \cdot 10^6$

to calculate the optical potential for the  $\alpha$ - $^8\text{Be}$  system using the folding procedure again. Since no experimental phase shifts for  $\alpha$ - $^8\text{Be}$  are available, we have to fit the normalization factors  $\lambda$  of the folding potentials to the bound and resonance-state energies of  $^{12}\text{C}$ .

The ground state ( $0_1^+$ ), the first ( $2_1^+$ ) and second ( $0_2^+$ ) excited, and the ( $2_2^+$ ) state in are those states which are of interest in the triple-alpha process (see Table 2). With respect to a  $\alpha \otimes ^8\text{Be}$  clustering in  $^{12}\text{C}$ , the  $0_1^+$  and  $2_1^+$  bound states belong to the ground state band with a harmonic oscillator quantum number  $Q=4$ , whereas for the  $0_2^+$ - and  $2_2^+$ -state we assume that they can be interpreted as members of a band with  $Q=6$ . We assume the broad 10.3 MeV level in  $^{12}\text{C}$  to be the  $2_2^+$  state. The adopted spin assignment of this level is  $0^+$  [10]. However, an assignment of  $2^+$  for this state is strongly suggested by experimental results [11] as well as by microscopic calculations [12].

**Table 3.**  $\Gamma_\gamma$ -widths and  $B(E2)$ -values

	$E_f$ [MeV]	$J_f^\pi$	$E_i$ [MeV]	$J_i^\pi$	$\Gamma_\gamma$ [meV]	$B(E2)$ [W.u.]
$^8\text{Be}$	0.0	$0^+$	3.04	$2^+$	8.4	75
$^{12}\text{C}$	4.437	$2_1^+$	7.654	$0_2^+$	4.1	8.8
	7.654	$0_2^+$	10.3	$2_2^+$	17	100

Using the double-folded  $\alpha$ - $^8\text{Be}$  potential we have fitted the energies of these  $^{12}\text{C}$  states. These states have quite a different  $\alpha\otimes^8\text{Be}(\text{g.s.})$  cluster structure as can be seen from the values of the spectroscopic factors:  $S(\text{g.s.})=0.557$  [8],  $S(2_1^+)=0.154$  [13], and  $S(^{12}\text{C}, 0_2^+)=1.8$  [14]. Therefore, the  $\alpha$ - $^8\text{Be}$  folding potentials have to be renormalized separately to the ground and excited states in  $^{12}\text{C}$ . The normalization factors  $\lambda$  for the potential obtained in this way are listed in Table 2. Using the optical model we also calculated the alpha widths  $\Gamma_\alpha$  of the resonant  $0_2^+$  and  $2_2^+$  states. The good agreement between the experimental and the calculated  $\alpha$ -width for the  $0_2^+$ -state confirms the assumption that this state has a marked  $\alpha\otimes^8\text{Be}(\text{g.s.})$ -structure.

Furthermore, we calculated the cross sections and astrophysical  $S$ -factors of the reactions  $^4\text{He}(\alpha, \gamma_0)^8\text{Be}$  and  $^8\text{Be}(\alpha, \gamma_1)^{12}\text{C}$  in the direct capture model [3], [15]. These cross sections are essentially determined by the overlap of the scattering wave functions in the entrance channel, the bound state wave functions in the exit channel, and the electric quadrupole operator. The wave functions were calculated using the  $\alpha$ - $\alpha$  and  $\alpha$ - $^8\text{Be}$  potential, respectively. In order to compute the reaction  $^4\text{He}(\alpha, \gamma_0)^8\text{Be}$  the resonance energy of the  $^8\text{Be}$  ground state was lowered by about 100 keV. This means that this state was assumed to be weakly bound. For the description of the bound states the  $\alpha$ -particle spectroscopic factors given above are used. In Fig. 2 the cross section of the resonant E2-capture into the  $^8\text{Be}$  ground state is shown. Excellent agreement is found with the result of a calculation in a bremsstrahlung model [16]. Approximating the  $\sigma(E)$  cross sections near the resonance energy by a Breit-Wigner parametrization,  $\Gamma_\gamma$ -widths and  $B(\text{E2})$ -values can be deduced. These quantities are given in Table 3.

Acknowledgments: We want to thank the DFG-project Sta290/2, and the Austrian Science Foundatio (FWF) (project P8806-PHY).

## References

- [1] Oberhummer, H., Krauss, H., Grün, K., Rauscher, T., Abele, H., Mohr, P., Staudt, G.: this volume
- [2] Kobos, A. M., Brown, B. A., Lindsay, R., Satchler, G. R.: Nucl. Phys. **A425**, 205 (1984)
- [3] Oberhummer, H., Staudt G.: in Nuclei in the Cosmos. Ed. H. Oberhummer, p. 29. Berlin: Springer-Verlag 1991
- [4] De Vries, H., Jager, C. W., de Vries, C.: At. Data and Nucl. Data Tables **36**, **495** (1987)
- [5] Abele H.: computer code DFOLD, Univ. Tübingen, unpublished

- [6] Abele H.: Ph.D. thesis, Univ. Tübingen 1992
- [7] Wildermuth, K., Tang, Y. C.: A Unified Theory of the Nucleus. Braunschweig: Vieweg & Son 1977
- [8] Kurath, D.: Phys. Rev. **C7**, 1390 (1973)
- [9] Ajzenberg–Selove, F.: Nucl. Phys. **A490**, 1 (1988)
- [10] Ajzenberg–Selove, F.: Nucl. Phys. **A506**, 1 (1990)
- [11] Jacquot, C., Satamoto, Y., Jung, M., Girardio, L.: Nucl. Phys. **201**, 247 (1973)
- [12] Fujiwara, Y. et al.: Prog. Theor. Phys. Suppl. **68**, 29 (1980)
- [13] Kwaśniewicz, E., Jarczyk, L.: J. Phys. **G12**, 697 (1986)
- [14] Fukushima, Y., Kamimura, N.: J. Phys. Soc. Japan **44**, 225 (1978)
- [15] Mohr, P., Abele, R., Zwiebel, G., Staudt, G., Krauss, H., Oberhummer, H. Denker A., Hammer J. W., Wolf, G.: Phys. Rev. C, in press
- [16] Langanke, K.-H., Rolfs, C.: Phys. Rev. **C33**, 790 (1986)

## Captions to figures

Fig. 1: Nucleon density distribution of  ${}^8\text{Be}$  calculated in the potential model

Fig. 2: Capture cross section of  ${}^4\text{He}(\alpha, \gamma){}^8\text{Be}$  near the  $2^+$  resonance